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*Published in:*  
Physics Letters B

*DOI:*  
[10.1016/0370-2693\(87\)91081-1](https://doi.org/10.1016/0370-2693(87)91081-1)

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*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
1987

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Wu, H. C., Dieperink, A. E. L., & Scholten, O. (1987). Magnetic dipole properties in the SU(3) limit of IBA-2 with L=0, 2 and 4 bosons. *Physics Letters B*, 187(3-4), 205-209. [https://doi.org/10.1016/0370-2693\(87\)91081-1](https://doi.org/10.1016/0370-2693(87)91081-1)

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## MAGNETIC DIPOLE PROPERTIES IN THE SU(3) LIMIT OF IBA-2 WITH $L=0, 2$ AND 4 BOSONS

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Received 2 October 1986; revised manuscript received 15 December 1986

Properties of the magnetic dipole operator in the neutron–proton IBA model with  $L=0, 2$  and 4 bosons are discussed. Analytic expressions are derived for M1 matrix elements for low-lying states in the SU(3) limit

One of the most interesting recent developments in nuclear spectroscopy has been the prediction [1] and experimental confirmation [2] of collective magnetic dipole strength in deformed nuclei in the rare-earth and transactinide transitional region. This strength of several s.p. units has been interpreted as an isovector orbital rather than a spin type of excitation.

While in the simplest approaches, e.g. the extreme Nilsson model with degenerate single-particle levels [3], or the SU(3) limit of the IBA-2 model [1], the M1 strength is predicted to be concentrated into one level, more realistic calculations in the framework of HFB plus particle–hole excitations tend to predict [4] fragmentation of the total M1 strength over a large energy interval (3–10 MeV). The recent analysis of ( $e, e'$ ) and ( $\gamma, \gamma'$ ) experiments seems to indicate that a substantial part of the orbital M1 strength is concentrated around 3 MeV with a  $B(M1)$  strength  $\sim 2\mu_N^2$  and a width of about 0.5 MeV [2].

In the original IBA-2 approach with  $L=0, 2$  bosons the M1 strength is predicted in the SU(3) limit [1]. The sum-rule strength agrees with the observed values for  $^{156}\text{Gd}$  [2] but for some other nuclei in this mass region (e.g.  $^{164}\text{Dy}$ ) it appears to underestimate the strength by about a factor 2 [5]. The observed fragmentation cannot be explained, even if an appreciable breaking of the SU(3) limit is allowed. This indicates that other degrees of freedom outside the

sd IBA-2 space play a role. Various microscopic calculations and analysis of the HFB ground state wave function [6] also suggest that there is a small but non-negligible contribution from the  $L=4$  pair ( $\sim 5$ –10%).

Although some of the effects of the g-pair can be absorbed into a renormalization of the parameters in the sd space there are also observables which depend explicitly on the  $L=4$  pair, e.g. the distribution of E4 strength and the fragmentation of M1 strength.

Some investigations of the effect of the  $L=4$  g-boson on observables has been reported already. Barrett and Halse [7] showed that in the SU(3) limit of IBA-2 the inclusion of the g-boson leads to a doubling of the  $B(M1)$  strength as compared to the sd space. Pittel et al. [8] performed a mean field plus TDA calculation for a broken SU(3) hamiltonian and found a strong fragmentation of the M1 strength, depending upon the choice of the parameters.

In this note we investigate to what extent the M1 strength distribution is modified by the inclusion of the  $L=4$  boson in IBA-2. The main feature is that in the sdg space there is a new rank-one operator, in addition to the total angular momentum in sd IBA. For the evaluation of the matrix elements we will use the SU(3) limit, appropriate for strongly deformed nuclei.

For an axially symmetric deformed nucleus the appropriate dynamic symmetry of the neutron–proton sdg IBA model is given by

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$$U^{(\pi)}(15) \times U^{(\nu)}(15) \supset U^{(\pi+\nu)}(15)$$

$$\begin{array}{ccc} | & | & | \\ [N_\pi] & [N_\nu] & [N, F] \\ \supset SU^{(\pi+\nu)}(3) \supset SO^{(\pi+\nu)}(3) \\ | & | & \\ (\lambda, \mu) & L & \end{array}$$

The allowed SU(3) irreps  $(\lambda, \mu)$  can be obtained from the branching rules

$$[N_\pi] \times [N_\nu] = \sum_{t=0}^{|N_\pi - N_\nu|} [N-t, t],$$

where

$$[N] = (4N, 0) + (4N-4, 2) + (4N-6, 3)$$

$$+ (4N-8, 4)^2 + \dots,$$

$$[N-1, 1] = (4N-2, 1) + (4N-4, 2)$$

$$+ (4N-6, 3)^2 + (4N-8, 4)^4 + \dots,$$

$$[N-2, 2] = (4N-4, 2) + \dots,$$

in which the superscripts describe degeneracy of the corresponding SU(3) irreps. The most general hamiltonian with this dynamic symmetry

$$H = \alpha C_2[SU_3] + \beta C_2[O(3)] + \gamma C_2[U(15)], \quad (1)$$

can be rewritten in terms of the more physical operators

$$H = \alpha' Q^{(2)} \cdot Q^{(2)} + \beta' L^{(1)} \cdot L^{(1)} + \gamma' \tilde{M}, \quad (2)$$

where  $\tilde{M}$  is the Majorana operator with a spectrum  $\langle \tilde{M} \rangle = F(F+1)$ . For  $\alpha' < 0$  the lowest symmetric representations ( $F = F_{\text{Max}} = \frac{1}{2}N$ ) are:  $(\lambda, \mu) = (4N, 0)$  (the ground state band),  $(4N-4, 2)$  ( $K=0, 2$ ) (the  $\beta$  and  $\gamma$  band) and  $(4N-6, 3)$  ( $K=1, 3$ ). In addition there are mixed neutron-proton symmetry states ( $F = \frac{1}{2}N-1$ ):  $(4N-2, 1)$  ( $K=1$ ) (the so-called "scissor mode"),  $(4N-4, 2)$ , ( $K=0, 2$ ) (mixed-symmetry  $\beta'$ ,  $\gamma'$  bands), and the bands that belong to the doubly degenerate  $(4N-6, 3)$  ( $K=1, 3$ ) representation.

Therefore in the lowest SU(3) irreps with  $\mu \leq 3$  and  $F \geq F_M - 1$ , there are four  $K=1$  bands. A schematic spectrum for those  $K^\pi = 1^+$  bandheads is shown in fig. 1.

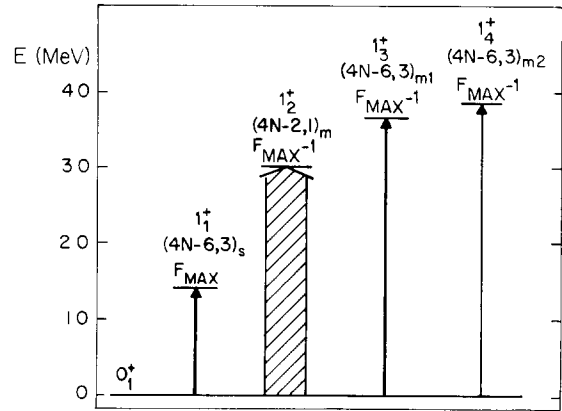


Fig. 1 A schematic spectrum for lowest  $1^+$  states in the sdg IBA-2 model. The arrows represent the strengths of M1 transition from ground state ( $0^+$ ) to the corresponding  $1^+$  states. The explanation of the subscripts for the SU(3) irreps can be found in the text.

In the present case there are four independent one-body rank-one operators, which are linear combinations of the generators  $L_{d,\rho}^{(1)} = \sqrt{10} (a_\rho^\dagger \tilde{a}_\rho)^{(1)}$  and  $L_{g,\rho}^{(1)} = \sqrt{60} (g_\rho^\dagger \tilde{g}_\rho)^{(1)}$  for neutrons and protons ( $\rho = \pi, \nu$ ). While one of these combinations is the total angular momentum  $L = L_d^{(1)} + L_g^{(1)} = L_\pi^{(1)} + L_\nu^{(1)}$ , the remaining three operators can be chosen in several convenient ways. For example, from the group theoretical point of view the operators  $L_a^{(1)}$  (see eq. (5)) and  $\sqrt{6} L_{d,\rho}^{(1)} - (1/\sqrt{6}) L_{g,\rho}^{(1)}$  form a natural choice since they transform like (1,1) and (3,3) representations of SU(3), respectively. For the physics it is more meaningful to construct rank-one operators that in the ground state band have no component in the direction of  $L$ . This leads to the following operators

$$A_\rho^{(1)} = \frac{1}{2} (4L_{d,\rho}^{(1)} - 3L_{g,\rho}^{(1)}), \quad \rho = \pi, \nu \quad (4)$$

and

$$L_a^{(1)} = \frac{1}{N} (N_\nu L_\pi^{(1)} - N_\pi L_\nu^{(1)}), \quad (5)$$

which satisfy

$$\begin{aligned} \langle \Phi_0 | A_\rho^{(1)} \cdot L^{(1)} | \Phi_0 \rangle \\ = \langle \Phi_0 | L_a^{(1)} \cdot L^{(1)} | \Phi_0 \rangle = 0, \end{aligned} \quad (5a)$$

where  $|\Phi_0\rangle$  is the SU(3) intrinsic state for the ground state band  $(\lambda, \mu) = (4N, 0)$  [9]:

$$|\Phi_0\rangle = \prod_{\rho} (\sqrt{1/5} s_{\rho}^{+} + \sqrt{4/7} d_{\rho,0}^{+} + \sqrt{8/35} g_{\rho,0}^{+})^{N_{\rho}} |0\rangle. \quad (6)$$

The most general one-body M1 operator can thus be expressed as

$$T_{\mu}(M1) = \sqrt{3/4\pi} \times \left( g_R L^{(1)} + (g_{R,\pi} - g_{R,\nu})^{(1)} + \sum_{\rho} h_{\rho} A_{\rho}^{(1)} \right), \quad (7)$$

where the bosonic gyromagnetic ratios are given by

$$g_R = \frac{1}{N} (N_{\pi} g_{R,\pi} + N_{\nu} g_{R,\nu}), \quad (8)$$

with

$$g_{R,\rho} = \frac{1}{2} (3g_{d,\rho} + 4g_{g,\rho}), \quad h_{\rho} = g_{d,\rho} - g_{g,\rho}, \quad (8a)$$

in which  $g_{d,\rho}$  and  $g_{g,\rho}$  are the gyromagnetic ratios of the d- and g-boson (for proton or neutron), respectively.

Clearly the first term in eq. (7) is diagonal in  $L$  and contributes to magnetic moments only. The second term, associated with the neutron-proton degree of freedom, has selection rules  $\Delta F=1$ ,  $\Delta\mu=1$  and connects the ground state band with the so-called scissor band; the third term with selection rules  $\Delta F=0,1$ ,  $\Delta\mu=1,3$  describes effects associated with the possible difference between the  $g$ -factors for  $L=2$  and  $L=4$  pairs. A microscopic estimate of the values of  $g_{R,\rho}$  and  $h_{\rho}$  for  $^{156}\text{Gd}$  will be given below.

The matrix elements of the operator (7) in the  $U(15) \supset SU(3) \supset O(3)$  basis  $\langle [N, F'] (\lambda', \mu') K' L' \| (b_1^\dagger b_\lambda)^{(1)} \| [N, F] (\lambda, \mu) KL \rangle$  can be obtained in closed form for the cases of interest by using the  $U(15) \supset SU(3)$  and  $SU(3) \supset O(3)$  isoscalar factors (ISF). A general method for the construction of intrinsic states (for the  $SU(3)$  limit) and the  $U(15) \supset SU(3)$  ISF has been discussed in ref. [9], and it is applied for the present sdg IBA-2 case.

To show the main feature of the M1 properties we restrict ourselves to the large  $N$  limit in which case the rotational motion can be factored out (for states with  $L \ll N_{\pi}, N_{\nu}$ ) and the  $SU(3) \supset O(3)$  ISF are not needed.

Of particular interest are the  $B(M1)$  values for the  $0^+$  ground state to the  $L^{\pi}=1^+$  bandheads for the various  $K=1$  bands<sup>†1</sup>:

$$B(M1, \text{g.s.} \rightarrow (4N-2, 1)_m L^{\pi}=1_2^+) = \frac{3}{4\pi} \frac{8N_{\pi}N_{\nu}}{N} 3(g_{R,\pi} - g_{R,\nu})^2, \quad (9a)$$

$$B(M1, \text{g.s.} \rightarrow (4N-6, 3)_s L^{\pi}=1_1^+) = \frac{3}{4\pi} \frac{96}{49} \frac{(N_{\pi}h_{\pi} + N_{\nu}h_{\nu})^2}{N}, \quad (9b)$$

$$B(M1, \text{g.s.} \rightarrow (4N-6, 3)_{m1} L^{\pi}=1_3^+) = \frac{3}{4\pi} \frac{192}{49} \frac{N_{\pi}N_{\nu}}{N} (h_{\pi} - h_{\nu})^2, \quad (9c)$$

$$B(M1, \text{g.s.} \rightarrow (4N-6, 3)_{m2} L^{\pi}=1_4^+) \approx \frac{1}{N} B(M1, \text{g.s.} \rightarrow (4N-6, 3)_{m1} L^{\pi}=1_3^+), \quad (9d)$$

where the subscripts  $s$  and  $m$  describe symmetric and mixed-symmetry irreps, and subscripts  $m1$  and  $m2$  correspond to single- and double-boson excitations [9], respectively.

The  $B(M1)$  values in the (symmetric)  $\gamma$ -band are

$$B(M1, \gamma L \rightarrow \gamma L') = \frac{3}{4\pi} \frac{16}{49} (2L' + 1) \times \left( \begin{matrix} L & 1 & L' \\ 2 & 0 & 2 \end{matrix} \right)^2 \left( \frac{h_{\pi}N_{\pi} + h_{\nu}N_{\nu}}{N} \right)^2. \quad (10)$$

By virtue of the condition (5a), only  $L^{(1)}$  gives a contribution to the magnetic moments of the levels in the ground state band and  $\beta$ -band,

$$\mu(L) = g_R L, \quad \text{g.s. band and } \beta\text{-band}. \quad (11a)$$

For the bands with  $K \neq 0$  the expectation value of the inproduct of  $L^{(1)}$  with the operators of eqs. (4), (5) do not vanish, thus giving rise to  $g$ -factors that differ from those of the ground state band,

<sup>†1</sup> Eq (9a) has been derived before by Barrett and Halse [7]

$\mu(L)$ 

$$= g_R L + \frac{4}{7} \frac{1}{N} (h_\pi N_\pi + h_\nu N_\nu) \frac{2}{L+1}, \quad \gamma\text{-band} . \quad (11b)$$

In contrast, in the sd IBA-2 model all the levels with the same value of  $F$ -spin have equal  $g$ -factors.

As is discussed in the following, the gyromagnetic ratios  $g_{R,\rho}$  are considerably larger than  $h\rho$ , therefore the M1 transition to the scissor state  $1_2^+$  is dominant, in a qualitative agreement with the experimental results [2]. From (9b) and (9c) one sees that these M1 transition strengths are related to the ratios  $h\rho$ , which describes the difference between the  $g$ -factors of  $L=2$  and  $L=4$  pairs. Since this difference finds its origin in the difference of the expectation value of the nucleon spin operator,  $s$ , for the d- and the g-boson states, the transitions to the non-scissor  $1^+$  states ( $1_1^+$ ,  $1_3^+$  and  $1_4^+$  in fig. 1) are dominated by the spin part of the nucleon M1 operator. We notice that the structure of the expressions (11), (12) for the  $\Delta K=0$  M1 matrix elements for the  $\gamma$ -band is similar to that of the geometric model [10] where the magnetic moment is decomposed in two parts: one is related to rotation of the nucleus, and the other the intrinsic contribution ( $g_K - g_R$ ), which can be considered as the counterpart of the second term in the r.h.s. of eq. (11b).

The boson  $g$ -factors in (7) can be regarded as the image of the  $g$ -factors of the corresponding fermion pair states. A simple way [11] to obtain a first order estimate for these boson  $g$ -factors is to equate the M1 matrix elements in the fermion and boson spaces for the special case of lowest seniority, and  $n_d$  and  $n_g$ , respectively, i.e. one equates ( $\rho = \pi, \nu$ )

$$\begin{aligned} & \langle S^{N-1} A \| T^{(F)}(M1) \| S^{N-1} A \rangle_\rho \\ & \equiv g_{b,\rho} \langle s^{N-1} b \| (b^\dagger \tilde{b})^{(1)} \| s^{N-1} b \rangle_\rho , \end{aligned} \quad (12)$$

where  $A$  represents a collective  $L=2$  or 4 fermion pair,  $b$  represents the corresponding boson, and the fermionic M1 operator is given by

$$T(M1) = \sqrt{\frac{3}{4\pi}} \sum_i (g_i l_i + g_s s_i) .$$

This procedure has only been shown to be good near the vibrational limit. For deformed nuclei the seniority scheme is broken and more configurations with higher seniority should be included. However, the OAI method in the generalized seniority scheme seems to give a reasonable approximation of the general trends for deformed nuclei [12]. Being encouraged by this, bearing in mind that the gyromagnetic ratios (related to odd rank tensor operators) are not sensitive to seniority mixing, we used this procedure to obtain a qualitative estimate of the  $g$ -factors for deformed nuclei. This has been done since a widely recognized quantitative microscopic theory for the SU(3) limit of the IBA is still absent. By using the method of ref. [12], we get the  $g$ -factors for  $^{156}\text{Gd}$  ( $N_\pi=7, N_\nu=5$ ):  $g_{d,\pi}=1.12$ ,  $g_{g,\pi}=1.23$ ,  $g_{d,\nu}=-0.06$ ,  $g_{g,\nu}=-0.03$  [ $\mu_N$ ] and thus  $g_R=0.67$  and  $h_\pi=-0.12$ ,  $h_\nu=-0.03$  [ $\mu_N$ ]. These values vary only slightly from isotope to isotope.

The important points to note are: (i) due to the collectivity of the pairs the net contribution from the spins  $\langle \sum_i s_i \rangle$  is small ( $\sim 10\%$ ) compared to the orbital proton contribution; (ii) since the  $L=2$  pair is in general more collective than the  $L=4$  pair the values of  $h_\pi, h_\nu$  do not vanish and are in the order of  $0.1\mu_N$ . The values for  $h_\pi, h_\nu \approx 0.1\mu_N$  seem not unreasonable; for example, they lead to  $B(M1)$  values for  $\gamma \rightarrow \gamma$  transitions of the order of  $10^{-2} [\mu_N^2]$  which agrees with typical experimental values [13] for deformed nuclei.

In summary we have shown that the inclusion of the g-boson in the IBA model gives rise to a fragmentation of isovector M1 strength in the SU(3) region, and an intrinsic contribution to M1 matrix elements between low-lying states. This extension of the model allows for a description of not only the orbital part of the M1 operator, but also gives some predictions for the spin part in the framework of the IBA model. Differences in  $g$ -factors are predicted, even for fully  $F$ -spin symmetric states.

This work was performed as part of the research program of "Stichting voor Fundamenteel Onderzoek der Materie" (FOM) which is financially supported by the "Organisatie voor Zuiver Wetenschappelijk Onderzoek" (ZWO). We also acknowledge the support by the NATO research grant RG 85/0036.

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